Chapter 13

An Introduction to Closure Phases

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13.1 Introduction

Phase distortions due to atmospheric turbulence, as emphasized by Quirrenbach in Chapter 5, cause a variety of ill effects. They both limit the maximum useful size of a collecting aperture and the longest allowed integration times, setting severe limitations on sensitivity. However even when a fringe is detected, the random and unknown atmospheric path delays cause phase shifts which erase information about the *intrinsic* phase arising from source structure. This chapter will discuss the use of *closure phases*, first invented for use in radio interferometry to recover most, if not essentially all, of this lost phase information for interferometric arrays with three or more telescopes.

13.1.1 Telescope Errors: Complex Gain

In an interferometric array, amplitude and phase errors associated with telescope i can be conceptualized in terms of a complex gain, $\tilde{G}_i$, where the tilde is used to indicate a complex number endowed with both an amplitude and phase. At radio wavelengths, the electric field of the incoming radiation can be directly measured at each telescope; in the visible/infrared, the field is not measured before interference, but rather is modified by the atmosphere and optics in each telescope before beam combination. In either case, the "measured" electric field can be represented as follows:

$$\begin{aligned}
\tilde{E}_i^{\text{measured}} &= \tilde{G}_i \tilde{E}_i^{\text{true}} && (13.1)\\
&= |G_i| e^{i\Phi_i^G} \tilde{E}_i^{\text{true}}. && (13.2)
\end{aligned}$$

The amplitude of $\tilde{G}$ corresponds to the overall scale factor, collectively representing all telescope-specific effects which modify the intensity of the received stellar radiation, e.g. mirror reflectivity, detector sensitivity, local scintillation. The phase Φ_i^G encodes all telescope-specific phase shifts, such as those due changing optical pathlengths from thermal expansion/contraction or *atmospheric turbulence* above the telescope.

How do such errors effect the measurement of the complex visibility? When light from two telescopes i and j are interfered, the visibility $\tilde{\mathcal{V}}_{ij}$ is derived from the contrast of the resulting fringes. Using Equation 13.2, we can see how the telescope-specific errors, represented by complex gains $\tilde{G}$, affect the measured visibility:

$$\begin{aligned}
\text{Since} \qquad \tilde{\mathcal{V}}_{ij} &\propto \tilde{E}_i \cdot \tilde{E}_j^*, && (13.3)\\
\tilde{\mathcal{V}}_{ij}^{\text{measured}} &= \tilde{G}_i \tilde{G}_j^* \tilde{\mathcal{V}}_{ij}^{\text{true}} && (13.4)\\
&= |G_i||G_j| e^{i(\Phi_i^G - \Phi_j^G)} \tilde{\mathcal{V}}_{ij}^{\text{true}}. && (13.5)
\end{aligned}$$

Atmospheric Phase Errors

From Equation 13.5, we can see that the measured phase of a detected fringe is shifted by the phase difference of the phase offsets at the individual telescopes. This can be easily seen in the idealized interferometer sketched in Figure 13.1. In this figure, an optical interferometer is represented by a Young's two-slit experiment (Born and Wolf 1999). Flat wavefronts from a distant source impinge on the slits and produce an interference pattern on an illuminated screen; this interference pattern drawn corresponds to the field *intensity*, not the electric field strength.

The spatial frequency of these (intensity) fringes is determined by the distance between the slits (in units of the wavelength of the illuminating radiation). However if the pathlength above one slit is changed (due to a pocket of warm air moving across the aperture, for example), the interference pattern will be shifted by an amount depending on the difference in pathlength of the two legs in this simple interferometer. If the extra pathlength is half the wavelength, the fringe pattern will shift by half a fringe, or π radians. The phase shift is completely independent of the slit separation, and only depends on slit-specific phase delays (as in Equation 13.5).

Why Not Average Phase?

One might think that the intrinsic phase of the interference pattern could be recovered by averaging over many realizations of the atmosphere. Even when the average atmosphere-

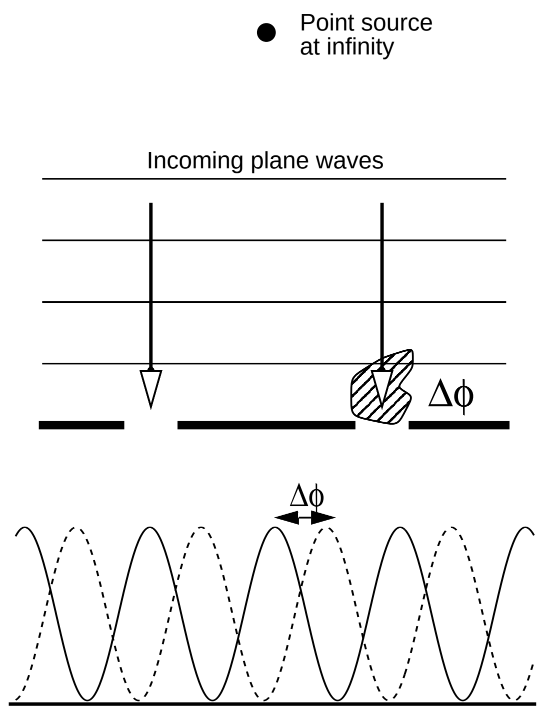

Figure 13.1: Phase errors at telescopes cause fringe shifts, as can be seen through analogy with Young's two-slit experiment.

induced phase shift is zero, the intrinsic phase can not be deduced if the rms phase shift $\Delta\phi_{\mathrm{atm}}$ is greater than about 1 radian.

This is illustrated in Figure 13.2. The top panel (case 1) shows the distribution of measured phases when $\Delta\phi_{\mathrm{atm}}$ is less than one radian, while the bottom panel (case 2) shows what happens when $\Delta\phi_{\mathrm{atm}} \gg 1$ radian. Clearly, unless one knows *which* fringe one is measuring, the phase wrapping induced by large atmospheric fluctuations completely scrambles the phase information on any given baseline. Under typical seeing conditions, one can expect pathlength fluctuations up to 5–10 μm, hence most optical and infrared optical interferometers operate under conditions similar to case 2, and baseline phase information is destroyed by the turbulent atmosphere.

The loss of this phase information has serious consequences. Imaging of non-centrosymmetric objects rely on the Fourier phase information encoded in this intrinsic phase of interferometer fringes. Without this information, imaging can not be done except for simple objects such as disks or round stars. Fortunately, a number of strategies have evolved to circumvent these difficulties.